Heavy quarkonium production through the top quark decays via flavor changing neutral currents

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(Dated: April 5, 2013)

Abstract

The production of the heavy charmonium and $(c\bar{b})$-quarkonium, through the top quark semi-exclusive decays via the flavor changing neutral currents (FCNC), has been systematically studied within the non-relativistic QCD. In different to the conventional squared amplitude approach, to simplify the results as much as possible, we adopt the 'improved trace technology' to do our calculation, which deals with the hard scattering directly at the amplitude level. If assuming the higher excited heavy-quarkonium states, such as the color-singlet and spin-triplet S-wave state $|[^3S_1]_{\mathbf{1}}\rangle$, the color-singlet P-wave states $|[^1P_1]_{\mathbf{1}}\rangle$ and $|[^3P_J]_{\mathbf{1}}\rangle$ (with $J=0,1,2$), and the two color-octet components $|[^1S_0]_{\mathbf{8}}g\rangle$ and $|[^3S_1]_{\mathbf{8}}g\rangle$, decay to the ground color-singlet and spin-singlet S-wave state $|[^1S_0]_{\mathbf{1}}\rangle$ with 100% efficiency via the electromagnetic or hadronic radiations, we obtain the total decay width: $\Gamma_{t\to|(c\bar{c})[^1S_0]_{\mathbf{1}}\rangle} = 171.1^{+147.7}_{-68.8}$ KeV and $\Gamma_{t\to|(c\bar{b})[^1S_0]_{\mathbf{1}}\rangle} = 7.32^{+2.49}_{-1.75}$ KeV, where the uncertainties are caused by varying $m_t = 172.0 \pm 4.0$ GeV, $m_b = 4.90 \pm 0.40$ GeV and $m_c = 1.50 \pm 0.25$ GeV. At the LHC with the center-of-mass energy $\sqrt{S} = 14$ TeV and a high luminosity $\mathcal{L} \propto 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$, sizable heavy-quarkonium events can be produced through the top quark decays via FCNC.

PACS numbers: 12.38.Bx, 14.65.Ha, 12.39.Jh, 14.40.Pq

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I. INTRODUCTION

With a high luminosity $\mathcal{L} \propto 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ and a high proton-proton collision energy $\sqrt{S} = 14$ TeV, a large amount of t-quark events about the order of 10^8 per year will be generated at the large hadronic collider (LHC) [1, 2]. This makes LHC a 'top factory' that shall provide us a unique platform to study the heavy quarkonium properties in comparison to the proton-anti-proton collider TEVATRON and the e^+e^- colliders such as Belle, BABAR, BEPC, CLEO-c and etc..

Being the heaviest fermion with a mass close to the electroweak symmetry breaking scale in the standard model (SM), the top quark is speculated to be a sensitive probe of new physics beyond the SM. In the literature, many studies have shown that the top quark rare decays via a flavor changing neutral current (FCNC), i.e. $t \to Vq$, $(V = \gamma, Z, g;\ q = c, u)$ [3, 4], can be significantly enhanced in some new physics models like the minimal supersymmetric model (MSSM) [5], the Topcolor-assisted Technicolor Model (TC2) [6] and other models (cf. Ref.[7]). Their results make the observation of any FCNC top quark processes a robust evidence for new physics.

On the other hand, a better understanding of these channels within the SM is helpful for judging whether there is really new physics, i.e. to deduct the SM background from the experimental data at a higher confidence level such that to determine the right ranges for the new physics parameters. Following the top quark dominant decay channel, $t \to bW^+$, it has been pointed out that sizable B_c^- mesons can be produced via the channel, $t \to |(b\bar{c})[n]\rangle + cW^+$ [8, 9], where $[n]$ stands for the dominant $(b\bar{c})$-quarkonium states via the velocity scaling rule of the non-relativistic QCD (NRQCD) [10]. It is interesting to show whether the rare SM FCNC channel $t \to |(c\bar{Q})[n]\rangle + QZ^0$ can also be a useful supplement for heavy quarkonium production, where Q stands for the heavy quark c or b. As a short notation, we represent the two heavy quarkoniums, i.e. the $(c\bar{c})$-charmonium and $(c\bar{b})$-quarkonium, as $(c\bar{Q})$-quarkonium.

In the NRQCD framework, a doubly heavy meson is considered as an expansion of various Fock states. In addition to the production of the two color-singlet S-wave states $|(c\bar{Q})[^1S_0]_{\mathbf{1}}\rangle$ and $|(c\bar{Q})[^3S_1]_{\mathbf{1}}\rangle$, a naive order estimation basing on the NRQCD velocity scaling rule [10] shows that the production of the four color-singlet P-wave states $|(c\bar{Q})[^1P_1]_{\mathbf{1}}\rangle$ and $(c\bar{Q})[^3P_J]_{\mathbf{1}}\rangle$ (with $J = 0, 1, 2$) together with the two color-octet components $|(c\bar{Q})[^1S_0]_{\mathbf{8}}g\rangle$

and $|(c\bar{Q})[^3S_1]_{\mathbf{8}}g\rangle$ shall also give sizable contributions. Here the thickened subscripts of $(c\bar{Q})$ denote the color indices, **1** for color singlet and **8** for color-octet; the relevant angular momentum quantum numbers are shown in the parentheses. It would be interesting to study various Fock states' contributions to make a sound estimation on the production of heavy quarkonium through the channel $t \to |(c\bar{Q})[n]\rangle + Z^0 Q$, and hence to be a useful reference for experimental studies.

The production of heavy quarkonium itself is very useful for testing perturbative QCD [11–14]. For example, in addition to the 'direct' hadronic production of B_c meson [15–19], it has been shown that its 'indirect' production channels via the top-quark, the Z^0-boson and the $W^{\pm}$-boson decays are important supplement and enough large events can also be produced at the LHC [8, 9, 20–26]. To make a systematic study on the B_c meson indirect production through the top quark decays via the rare FCNC channel is one of the purpose of the present paper.

The organization of the paper is as follows: In Sec.II, we describe our calculation technology for the top quark semi-exclusive decays to the heavy quarkonium via FCNC. Then we present numerical results and make some discussions on the properties of the heavy quarkonium production through the top quark decays in Sec.III. The final section is reserved for a summary.

II. TOP QUARK DECAYS VIA FCNC TO THE HEAVY QUARKONIUM

According to the NRQCD factorization formula [10], the decay width for the heavy quarkonium production via FCNC, i.e. $t(k) \to |(c\bar{Q})[n]\rangle(q_3) + Z^0(q_2) + Q(q_1)$, can be factorized as

$$d\Gamma = \sum_n d\hat{\Gamma}(t \to (c\bar{Q})[n] + Q + Z^0)\langle \mathcal{O}^H(n)\rangle, \tag{1}$$

where the non-perturbative matrix element $\langle \mathcal{O}^H(n)\rangle$ describes the hadronization of a perturbative $(c\bar{Q})$ pair with quantum number $[n]$ into the observable hadronic state H. The color-octet matrix elements are to be determined experimentally, which are smaller than the color-singlet matrix elements by certain v^2 order and can be estimated by the NRQCD velocity scaling rules. As for the color-singlet components, the matrix elements can be directly related to the wave functions at the origin for the S-wave states or the first derivative of the wave functions at the origin for the P-wave states, which can be computed through the

potential models, cf. Refs.[27–32].

The short-distance decay width

$$d\hat{\Gamma}(t \to (c\bar{Q})[n] + Q + Z^0)) = \frac{1}{2k^0}\overline{\sum}|M|^2 d\Phi_3, \tag{2}$$

where $\overline{\sum}$ means that we need to average over the spin and color states of the initial particles and to sum over the color and spin of all the final particles.

In the top quark rest frame, the three-particle phase space can be written as

$$d\Phi_3 = (2\pi)^4 \delta^4 \left(k - \sum_f^3 q_f\right) \prod_{f=1}^3 \frac{d^3\vec{q}_f}{(2\pi)^3 2q_f^0}. \tag{3}$$

The $1 \to 3$ phase space with the final massive particles can be found in appendix of Refs. [9, 21]. Here the parameters k and q_f are four-momenta of the corresponding particles.

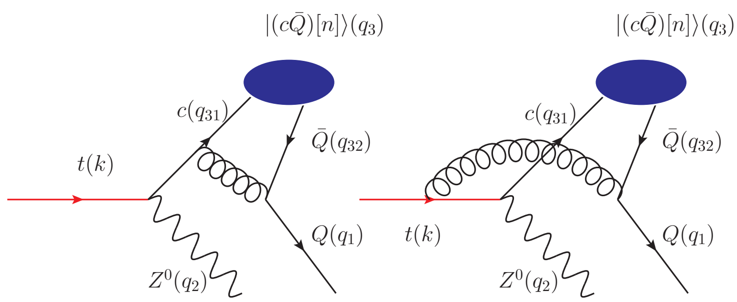

FIG. 1: Feynman diagrams for $t(k) \to |(c\bar{Q})[n]\rangle(q_3) + Z^0(q_2) + Q(q_1)$, where $|(c\bar{Q})[n]\rangle$ stands for the eight quarkonium Fock states $|(c\bar{Q})[^1S_0]_{\mathbf{1}}\rangle$, $|(c\bar{Q})[^3S_1]_{\mathbf{1}}\rangle$, $|(c\bar{Q})[^1P_1]_{\mathbf{1}}\rangle$, $|(c\bar{Q})[^3P_J]_{\mathbf{1}}\rangle$, $|(c\bar{Q})[^1S_0]_{\mathbf{8}}g\rangle$ and $|(c\bar{Q})[^3S_1]_{\mathbf{8}}g\rangle$, respectively.

As a short notation, we take $t(k) \to |(c\bar{Q})[n]\rangle(q_3) + Z^0(q_2) + Q(q_1)$ to stand for the semi-inclusive channels: $t \to |(c\bar{c})[n]\rangle + cZ^0$ and $t \to |(c\bar{b})[n]\rangle + bZ^0$. The decay-width of these processes can be written as:

$$d\Gamma \;=\; \frac{\langle \mathcal{O}^H(n)\rangle}{256\pi^3 m_t^3}\overline{\sum}|M|^2 ds_1 ds_2, \tag{4}$$

where m_t is the top quark mass, and the invariant masses $s_1 = (q_1 + q_3)^2$ and $s_2 = (q_1 + q_2)^2$. The integration over s_1 and s_2 can be done with the help of the VEGAS program [33] whose improved version can be found in the programs BCVEGPY [19] and GENXICC [34]. The symbol $\overline{\sum}$ means that we need to average over the color and spin states of the initial particles and to sum over the color and spin of all the final particles. With the help of the

formulas listed in [9, 21], one can not only derive the whole decay width but also obtain the corresponding differential decay widths that are helpful for experimental studies, such as $d\Gamma/ds_1$, $d\Gamma/ds_2$, $d\Gamma/d\cos\theta_{13}$ and $d\Gamma/d\cos\theta_{12}$, where $s_1=(q_1+q_3)^2$, $s_2=(q_1+q_2)^2$, θ_{13} is the angle between $\vec{q}_1$ and $\vec{q}_3$, and θ_{12} is the angle between $\vec{q}_1$ and $\vec{q}_2$.

Our remaining task is to deal with the hard-scattering amplitude for the specified processes

$$t \to (c\bar{c})[n] + cZ^0 \;\text{ and }\; t \to (c\bar{b})[n] + bZ^0. \tag{5}$$

These amplitudes can be generally expressed as

$$iM_{ss'} = \mathcal{C}\;\bar{u}_{si}(q_1)\sum_{n=1}^{m}\mathcal{A}_n u_{s'j}(k), \tag{6}$$

where $m=2$ stands for the number of Feynman diagrams of these two processes, s and s' are spin states, i and j are color indices for the outing Q quark and the initial top quark, respectively. The Feynman diagram of the process is presented in Fig.(1), where the intermediate gluon should be hard enough to produce a $c\bar{c}$ pair or a $b\bar{b}$ pair, so the amplitude is pQCD calculable. The overall factor $\mathcal{C}=\mathcal{C}_s$ or $\mathcal{C}_o$ stands for the specified quarkonium in color-singlet and color-octet, respectively. $\mathcal{C}_s = \frac{gg_s^2}{3\sqrt{3}\cos(\theta_W)}\delta_{ij}$ and $\mathcal{C}_o = \frac{gg_s^2}{4\cos(\theta_W)}(\sqrt{2}T^aT^bT^a)_{ij}$, where $\sqrt{2}T^b$ stands for the color of the color-octet quarkonium state and θ_W is the Weinberg angle. The amplitude $\mathcal{A}_n$ for each hadronic state can be read out from these Feynman diagrams and we put them in the appendix.

Because of the emergence of the massive-fermion lines, the analytical expression for the squared amplitude becomes too complex and lengthy. We adopt the 'improved trace technology' to simplify the amplitudes $M_{ss'}$ at the amplitude level. In different to the helicity amplitude approach, which is to get the numerical (complex) value for the whole amplitude at the amplitude level [19, 35–37], only the coefficients of the basic Lorentz structures are numerical at the amplitude level for the 'improved trace technology'. Because the basic Lorentz structures are limited, by using the 'improved trace technology', one can get the analytic expressions for the amplitude and conveniently result in the numerical value for the squared amplitude, thus, the numerical efficiency can also be greatly improved.

The 'improved trace technology' has been suggested and developed by Refs. [9, 20–24]. As an explanation of the approach, we first arrange the amplitude $M_{ss'}$ into four orthogonal sub-amplitudes M_i according to the spins of the outgoing quark Q with spin s and the top quark with spin s', then transform these sub-amplitudes into a trace form by properly

dealing with the massive spinors with the help of an arbitrary light-like momentum k_0 and an arbitrary space-like momentum k_1, which satisfies $k_1^2 = -1$ and $k_0 \cdot k_1 = 0$. The final results should be independent of k_0 and k_1, which provides a way to check the rightness of the derived results. One can choose them to be those that can maximally simply the amplitude. Then we do the trace of the Dirac γ-matrix strings at the amplitude level, which finally result in analytic series over some independent Lorentz-structures.

Following the standard procedures of the 'improved trace technology', we can derive the independent Lorentz-structures together with their coefficients for each hadronic state $|(c\bar{Q})[n]\rangle$. Here to short the paper, we will not present them here. The interesting reader may consult Refs.[9, 20–24] for detailed technology. The mathematical program for these production channels are available upon request.

III. NUMERICAL RESULTS

We adopt the following values to do the numerical calculation: $m_Z = 91.1876$GeV, $m_t = 172.0$GeV, $m_c = 1.50$GeV and $m_b = 4.90$GeV. Leading order α_s running is adopted and we set the renormalization scale to be $2m_c$, which leads to $\alpha_s(2m_c) = 0.26$. The color-singlet non-perturbative matrix elements can be related to the wave function at the origin $\Psi_S(0) = \sqrt{1/4\pi}R_S(0)$ and the first derivative of the wave function at the origin $\Psi'_P(0) = \sqrt{3/4\pi}R'_P(0)$, and we adopt [32]

$$|R_S(c\bar{c})(0)|^2 = 0.810 \text{ GeV}^3, \quad |R'_P(c\bar{c})(0)|^2 = 0.075 \text{ GeV}^5,$$
$$|R_S(c\bar{b})(0)|^2 = 1.642 \text{ GeV}^3, \quad |R'_P(c\bar{b})(0)|^2 = 0.201 \text{ GeV}^5.$$

For the color-octet S-wave matrix elements, based on the velocity scaling rule and under the vacuum-saturation approximation, we have [10, 11, 38]

$$\langle (c\bar{Q})[^1S_0]_{\mathbf{8}}|\mathcal{O}_{\mathbf{8}}(^1S_0)|(c\bar{Q})[^1S_0]_{\mathbf{8}}\rangle \simeq \Delta_S(v)^2 \langle (c\bar{Q})[^1S_0]_{\mathbf{1}}|\mathcal{O}_{\mathbf{1}}(^1S_0)|(c\bar{Q})[^1S_0]_{\mathbf{1}}\rangle, \tag{7}$$

and

$$\langle (c\bar{Q})[^3S_1]_{\mathbf{8}}|\mathcal{O}_{\mathbf{8}}(^3S_1)|(c\bar{Q})[^3S_1]_{\mathbf{8}}\rangle \simeq \Delta_S(v)^2 \langle (c\bar{Q})[^3S_1]_{\mathbf{1}}|\mathcal{O}_1(^3S_1)|(c\bar{Q})[^3S_1]_{\mathbf{1}}\rangle \tag{8}$$

with $\Delta_S(v) \sim v^2$.

A. Basic numerical results

As a reference, we calculate the decay width for the process $t \to c + Z^0$, which can be written as

$$\Gamma = \frac{G_F m_W^2 |\vec{p}| \sec^2(\theta_W)}{36\sqrt{2}\pi m_t m_Z^2} \left\{ 9 \left[3m_Z^2 \sqrt{{m_c}^2 + |\vec{p}|^2} + 2|\vec{p}|^2 \left(\sqrt{{m_c}^2 + |\vec{p}|^2} + \sqrt{m_Z^2 + |\vec{p}|^2} \right) \right] - \right.$$
$$8 \left[2\cos(2\theta_W) + 1 \right] \sin^2(\theta_W) \left[-3m_c m_Z^2 + 3m_Z^2 \sqrt{{m_c}^2 + |\vec{p}|^2} + \right.$$
$$\left.\left. 2|\vec{p}|^2 \left(\sqrt{{m_c}^2 + |\vec{p}|^2} + \sqrt{m_Z^2 + |\vec{p}|^2} \right) \right] \right\}, \qquad (9)$$

where $\vec{p}$ stands for the relative momentum between the final two particles in the top-quark rest frame

$$|\vec{p}| = \frac{\sqrt{[m_t^2 - (m_c - m_Z)^2][m_t^2 - (m_c + m_Z)^2]}}{2m_t}.$$

Then, we obtain $\Gamma_{t \to c + Z^0} = 0.395$ GeV.

$t \to \vert(c\bar{c})[n]\rangle$	Γ (KeV)	$\frac{\Gamma_{t\to\vert(c\bar{c})[n]\rangle}}{\Gamma_{t\to c+Z^0}}$
$t \to \eta_c$	70.2	1.78×10^{-4}
$t \to J/\psi$	72.0	1.82×10^{-4}
$t \to h_c$	9.01	2.28×10^{-5}
$t \to \chi_{c0}$	7.18	1.82×10^{-5}
$t \to \chi_{c1}$	8.05	2.04×10^{-5}
$t \to \chi_{c2}$	3.00	7.59×10^{-6}
$t \to \vert(c\bar{c})[^1S_0]_{\mathbf{8}} g\rangle$	$8.78v^4$	$2.22v^4 \times 10^{-5}$
$t \to \vert(c\bar{c})[^3S_1]_{\mathbf{8}} g\rangle$	$9.00v^4$	$2.28v^4 \times 10^{-5}$

TABLE I: Decay widths and branching fractions for the charmonium production through the channel $t \to |(c\bar{c})[n]\rangle + cZ^0$.

Total decay widths and their branching fractions for the channels $t \to |(c\bar{Q})[n]\rangle + QZ^0$ are listed in Tables I and II. The squared amplitude for the color-octet decay width is suppressed by eight times to that of the color-singlet case. As a combined effect of such color suppression and the relative velocity suppression (v^4-suppression at the squared amplitude level), the color-octet channels are quite small in comparison to their corresponding color-singlet production channels. So in the following discussion, if not specially stated, we shall not include the color-octet states' contributions into our discussions.

$t \to \lvert(c\bar{b})[n]\rangle$	Γ (KeV)	$\frac{\Gamma_{t\to\lvert(c\bar{b})[n]\rangle}}{\Gamma_{t\to c+Z^0}}$
$t \to B_c$	3.54	8.96×10^{-6}
$t \to B_c^*$	2.97	7.52×10^{-6}
$t \to h_b$	0.20	5.10×10^{-7}
$t \to \chi_{b0}$	0.34	8.71×10^{-7}
$t \to \chi_{b1}$	0.22	5.65×10^{-7}
$t \to \chi_{b2}$	0.02	3.84×10^{-8}
$t \to \lvert(c\bar{b})[^1S_0]_{\mathbf{8}}g\rangle$	$0.44v^4$	$1.12v^4 \times 10^{-6}$
$t \to \lvert(c\bar{b})[^3S_1]_{\mathbf{8}}g\rangle$	$0.37v^4$	$0.94v^4 \times 10^{-6}$

TABLE II: Decay widths and branching fractions for the $(c\bar{b})$-quarkonium production through the channel $t \to \lvert(c\bar{b})[n]\rangle + bZ^0$.

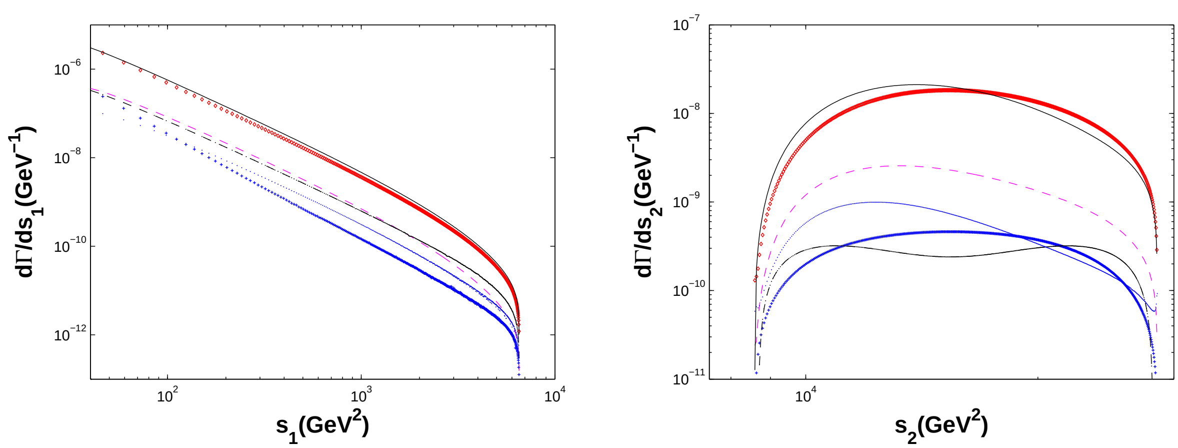

FIG. 2: Differential decay widths $d\Gamma/ds_1$ (Left) and $d\Gamma/ds_2$ (Right) for $t \to \lvert(c\bar{c})[n]\rangle + cZ^0$, where the diamond, the solid, the dashed, the crossed, the dash-dot and the dotted lines are for $\lvert(c\bar{c})[^1S_0]_{\mathbf{1}}\rangle$, $\lvert(c\bar{c})[^3S_1]_{\mathbf{1}}\rangle$, $\lvert(c\bar{c})[^1P_1]_{\mathbf{1}}\rangle$, $\lvert(c\bar{c})[^3P_0]_{\mathbf{1}}\rangle$, $\lvert(c\bar{c})[^3P_1]_{\mathbf{1}}\rangle$ and $\lvert(c\bar{c})[^3P_2]_{\mathbf{1}}\rangle$, respectively.

For the charmonium production channel $t \to \lvert(c\bar{c})[n]\rangle + cZ^0$, the total decay width for all the P-wave states is ~ 27.2 KeV, which is about 39% (38%) of that of η_c (J/ψ). For the $(c\bar{b})$-quarkonium production channel $t \to \lvert(c\bar{b})[n]\rangle + bZ^0$, the total decay width for all P-wave states is ~ 0.78 KeV, which is about 22% (26%) of that of B_c (B_c^*). This shows that the color-singlet P-wave states can have sizable contributions to the heavy quarkonium production.

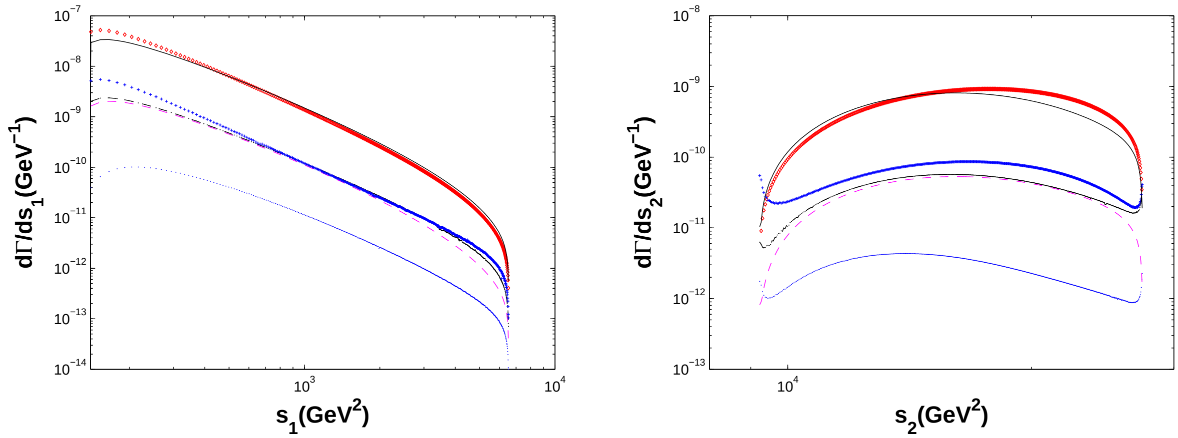

FIG. 3: Differential decay width $d\Gamma/ds_1$ (Left) and $d\Gamma/ds_2$ (Right) for $t \to |(c\bar{b})[n]\rangle + bZ^0$, where the diamond, the solid, the dashed, the crossed, the dash-dot and the dotted lines are for $|(c\bar{b})[^1S_0]_{\mathbf{1}}\rangle$, $|(c\bar{b})[^3S_1]_{\mathbf{1}}\rangle$, $|(c\bar{b})[^1P_1]_{\mathbf{1}}\rangle$, $|(c\bar{b})[^3P_0]_{\mathbf{1}}\rangle$, $|(c\bar{b})[^3P_1]_{\mathbf{1}}\rangle$ and $|(c\bar{b})[^3P_2]_{\mathbf{1}}\rangle$, respectively.

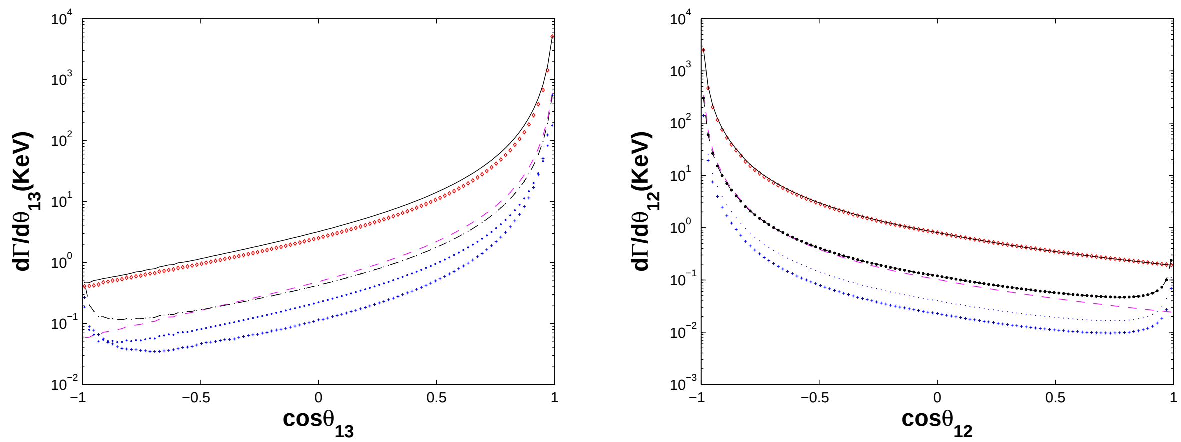

FIG. 4: Differential decay width $d\Gamma/d\cos\theta_{13}$ (Left) and $d\Gamma/d\cos\theta_{12}$ (Right) for $t \to |(c\bar{c})[n]\rangle + cZ^0$, where the diamond, the solid, the dashed, the crossed, the dash-dot and the dotted lines are for $|(c\bar{c})[^1S_0]_{\mathbf{1}}\rangle$, $|(c\bar{c})[^3S_1]_{\mathbf{1}}\rangle$, $|(c\bar{c})[^1P_1]_{\mathbf{1}}\rangle$, $|(c\bar{c})[^3P_0]_{\mathbf{1}}\rangle$, $|(c\bar{c})[^3P_1]_{\mathbf{1}}\rangle$ and $|(c\bar{c})[^3P_2]_{\mathbf{1}}\rangle$, respectively.

Further more, to show the relative importance among different Fock states, we present the differential distributions $d\Gamma/ds_1$ and $d\Gamma/ds_2$ in Figs.(2,3), and the differential distributions $d\Gamma/d\cos\theta_{13}$ and $d\Gamma/d\cos\theta_{12}$ in Figs.(4,5). Two invariant variables, $s_1 = (q_1 + q_3)^2$ and $s_2 = (q_1 + q_2)^2$, and θ_{13} stands for the angle between $\vec{q}_1$ and $\vec{q}_3$, θ_{12} stands for the angle between $\vec{q}_1$ and $\vec{q}_2$. The curves for $|(c\bar{Q})[^1S_0]_{\mathbf{1}}\rangle$, $|(c\bar{Q})[^3S_1]_{\mathbf{1}}\rangle$, $|(c\bar{Q})[^1P_1]_{\mathbf{1}}\rangle$ and $|(c\bar{Q})[^3P_J]_{\mathbf{1}}\rangle$ are presented. The difference between the color-singlet S-wave states and the color-octet

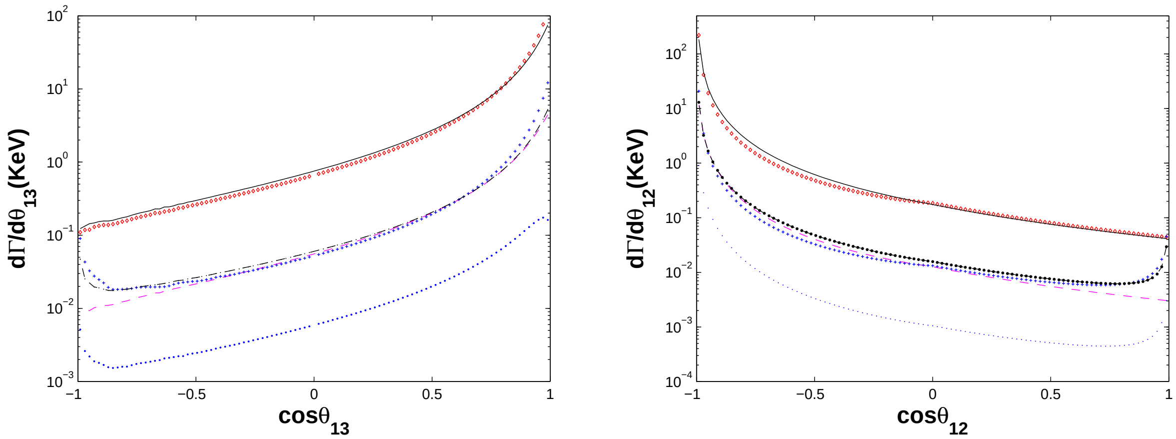

FIG. 5: Differential decay width $d\Gamma/d\cos\theta_{13}$ (Left) and $d\Gamma/d\cos\theta_{12}$ (Right) for $t \to |(c\bar{b})[n]\rangle + bZ^0$, where the diamond, the solid, the dashed, the crossed, the dash-dot and the dotted lines are for $|(c\bar{b})[^1S_0]_{\mathbf{1}}\rangle$, $|(c\bar{b})[^3S_1]_{\mathbf{1}}\rangle$, $|(c\bar{b})[^1P_1]_{\mathbf{1}}\rangle$, $|(c\bar{b})[^3P_0]_{\mathbf{1}}\rangle$, $|(c\bar{b})[^3P_1]_{\mathbf{1}}\rangle$ and $|(c\bar{b})[^3P_2]_{\mathbf{1}}\rangle$, respectively.

S-wave states is an overall color factor times the difference between the color-singlet and color-octet matrix elements, the shape of their curves are the same, so the curves of the color-octet ones are not presented in those figures.

As for the decay channels $t \to |(c\bar{c})[n]\rangle + cZ^0$ and $t \to |(c\bar{b})[n]\rangle + bZ^0$, because the heavy quarks and the heavy quarkoniums are much lighter than the Z^0 boson, the largest $d\Gamma/d\cos\theta_{13}$ is achieved when the heavy quarkonium and the outgoing heavy quark moving in the same direction ($\theta_{13} = 0^\circ$) or the Z^0 boson and the outgoing heavy quark moving back to back ($\theta_{12} = 180^\circ$), which are shown in Figs.(4, 5). This shows that the maximum differential decay width is obtained when the quarkonium and the outgoing heavy quark moving in the same direction or the heavy quarkonium and the outgoing Z^0 boson moving back to back in the rest frame of top quark.

Considering the LHC runs at the center-of-mass energy $\sqrt{S} = 14$ TeV with the luminosity $10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$, we can estimate the heavy-quarkonium events through the top quark decays via FCNC, i.e. about 1.8×10^4 η_c, 1.9×10^4 J/Ψ and 0.7×10^4 P-wave charmonium events per year can be generated; about 8.3×10^2 B_c, 7.5×10^2 B_c^* and 2.0×10^2 P-wave $(c\bar{b})$-quarkonium events per year can be generated. It might be possible to find J/ψ and B_c through top quark decays, since one may identify these particles through their cascade decay channels, $J/\psi \to \mu^+\mu^-$ and $B_c \to J/\psi + \pi$ or $B_c \to J/\psi + e\nu_e$ with clear signal, as well as Z^0 decays to a fair of leptons, $Z^0 \to l^+l^-$, where l stands for leptons, i.e. e, μ, τ.

Baring the situation pointed out here and the possible upgrade for the LHC (SLHC, DLHC and etc. [39]) in mind, the possibility to study the charmonium and the $(c\bar{b})$-quarkonium via top quark decays is worth seriously thinking about.

B. Uncertainty analysis

In this subsection, we shall discuss the uncertainties for the charmonium and the $(c\bar{b})$-quarkonium production through the top quark decays. At the present leading-order calculation, their main uncertainty sources include the non-perturbative bound state matrix elements, the renormalization scale μ_R and the quark masses m_t, m_b and m_c. The scale uncertainty can be suppressed by a higher-order perturbative calculation, or be solved by a proper scaling setting methods such as the recently suggested principle of maximum conformality (PMC) [40], in which the most important thing is to pick out the non-conformal β-series from higher order terms and to absorb them into the effective PMC scales, thus, the resulting series is conformal which is renormalization scheme and scale independent.

In the following, we shall concentrate our attention on the uncertainties caused by m_t, m_b and m_c. We let them within the range of $m_t = 172.0 \pm 4.0$ GeV, $m_b = 4.90 \pm 0.40$ GeV and $m_c = 1.50 \pm 0.25$ GeV. When discussing the uncertainty caused by one parameter, the other parameters are fixed to be their central values.

m_c(GeV)	1.25	1.50	1.75
$\Gamma_{\lvert(c\bar{c})[^1S_0]_{\mathbf{1}}\rangle}$(KeV)	122.2	70.2	43.9
$\Gamma_{\lvert(c\bar{c})[^3S_1]_{\mathbf{1}}\rangle}$(KeV)	125.5	72.0	44.9
$\Gamma_{\lvert(c\bar{c})[P-wave]_{\mathbf{1}}\rangle}$(KeV)	68.2	27.2	12.5
$\Gamma_{\lvert(c\bar{b})[^1S_0]_{\mathbf{1}}\rangle}$(KeV)	3.54	3.54	3.53
$\Gamma_{\lvert(c\bar{b})[^3S_1]_{\mathbf{1}}\rangle}$(KeV)	2.92	2.97	3.01
$\Gamma_{\lvert(c\bar{b})[P-wave]_{\mathbf{1}}\rangle}$(KeV)	1.05	0.78	0.62

TABLE III: Uncertainties for the decay width of the process $t \to |(c\bar{c})[n]\rangle + cZ^0$ and $t \to |(c\bar{b})[n]\rangle + bZ^0$ by varying $m_c \in [1.25, 1.75]$ GeV, where $|(c\bar{c})[P-wave]_{\mathbf{1}}\rangle$ and $|(c\bar{b})[P-wave]_{\mathbf{1}}\rangle$ stands for the sum of the four color-singlet P-wave states for the $(c\bar{c})$ and $(c\bar{b})$ quarkoniums respectively.

Typical uncertainties for m_c, m_b and m_t are presented in Tables III, IV and V, respectively.

m_b (GeV)	4.50	4.90	5.30
$\Gamma_{\|(c\bar{b})[^1S_0]_{\mathbf{1}}\rangle}$(KeV)	4.65	3.54	2.74
$\Gamma_{\|(c\bar{b})[^3S_1]_{\mathbf{1}}\rangle}$(KeV)	3.95	2.97	2.28
$\Gamma_{\|(c\bar{b})[P-wave]_{\mathbf{1}}\rangle}$(KeV)	1.06	0.78	0.59

TABLE IV: Uncertainties for the decay width of the process $t \to (c\bar{b})[n]$ by varying $m_b \in [4.50, 5.30]$ GeV, where $|(c\bar{b})_{\mathbf{1}}[P - wave]\rangle$ stands for the sum of the four color-singlet P-wave states.

m_t(GeV)	168.0	172.0	176.0
$\Gamma_{\|(c\bar{c})[^1S_0]_{\mathbf{1}}\rangle}$(KeV)	63.9	70.2	76.8
$\Gamma_{\|(c\bar{c})[^3S_1]_{\mathbf{1}}\rangle}$(KeV)	65.6	72.0	78.7
$\Gamma_{\|(c\bar{c})[P-wave]_{\mathbf{1}}\rangle}$(KeV)	25.3	27.2	29.2
$\Gamma_{\|(c\bar{b})[^1S_0]_{\mathbf{1}}\rangle}$(KeV)	3.18	3.54	3.91
$\Gamma_{\|(c\bar{b})[^3S_1]_{\mathbf{1}}\rangle}$(KeV)	2.68	2.97	3.27
$\Gamma_{\|(c\bar{b})[P-wave]_{\mathbf{1}}\rangle}$(KeV)	0.72	0.78	0.84

TABLE V: Uncertainties for the decay width of the process $t \to (c\bar{c})[n]$ and $t \to (c\bar{b})[n]$ by varying $m_t \in [168.0, 176.0]$ GeV, where $|(c\bar{c})[P - wave]_{\mathbf{1}}\rangle$ and $|(c\bar{b})[P - wave]_{\mathbf{1}}\rangle$ stands for the sum of the four color-singlet P-wave states for $(c\bar{c})$ and $(c\bar{b})$ quarkonium accordingly.

It shows that sizable uncertainties can be found for varying m_c, m_b and m_t, respectively. The decay width will decrease with the increment of m_c, m_b and m_t, and such tendency slowly down with a heavier quark mass. Moreover, one may observe that the decay widths for P-wave states are more sensitive to the quark masses than the case of S-wave states. Adding all the uncertainties in quadrature, we obtain

$$\begin{aligned}
\Gamma_{|(c\bar{c})[^1S_0]_{\mathbf{1}}\rangle} &= 70.2^{+52.0}_{-26.4}\ \text{KeV}, \\
\Gamma_{|(c\bar{c})[^3S_1]_{\mathbf{1}}\rangle} &= 72.0^{+53.6}_{-27.1}\ \text{KeV}, \\
\Gamma_{|(c\bar{c})[P-wave]_{\mathbf{1}}\rangle} &= 27.2^{+40.9}_{-14.7}\ \text{KeV}, \\
\Gamma_{|(c\bar{c})[S-wave]_{\mathbf{8}g}\rangle} &= 17.8^{+13.2}_{-6.70} \times v^4\ \text{KeV}. \qquad (10)
\end{aligned}$$

for $t \to |(c\bar{c})[n]\rangle + cZ^0$; and

$$\Gamma_{|(c\bar{b})[^1S_0]_{\mathbf{1}}\rangle} = 3.54^{+3.34}_{-2.38}\ \text{KeV},$$

$$\Gamma_{|(c\bar{b})[^3S_1]_{\mathbf{1}}\rangle} = 2.97^{+2.95}_{-2.08} \text{ KeV},$$
$$\Gamma_{|(c\bar{b})[P-wave]_{\mathbf{1}}\rangle} = 0.78^{+1.14}_{-0.76} \text{ KeV},$$
$$\Gamma_{|(c\bar{b})[S-wave]_{\mathbf{8}}g\rangle} = 0.81^{+0.26}_{-0.19} \times v^4 \text{ KeV}. \tag{11}$$

For $t \to |(c\bar{b})[n]\rangle + bZ^0$.

If assuming the high excited heavy-quarkonium states decay to the ground color-singlet and spin-singlet S wave state $|(c\bar{Q})[^1S_0]_{\mathbf{1}}\rangle$ with 100% efficiency via the electromagnetic or the hadronic interactions, then we obtain the total decay width of top quark decay channels

$$\Gamma_{(t\to|(c\bar{c})[^1S_0]_{\mathbf{1}}\rangle+cZ^0)} = 171.1^{+147.7}_{-68.8} \text{ KeV} \tag{12}$$
$$\Gamma_{(t\to|(c\bar{b})[^1S_0]_{\mathbf{1}}\rangle+bZ^0)} = 7.32^{+2.49}_{-1.75} \text{ KeV} \tag{13}$$

where we have set $v^2 = 0.3$ for the color-octet S-wave charmonium states $|(c\bar{c})[^1S_0]_{\mathbf{8}}g\rangle$ and $|(c\bar{c})[^3S_1]_{\mathbf{8}}g\rangle$, and $v^2 = 0.2$ for the color-octet S-wave $(c\bar{b})$-quarkonium states $|(c\bar{b})[^1S_0]_{\mathbf{8}}g\rangle$ and $|(c\bar{b})[^3S_1]_{\mathbf{8}}g\rangle$, respectively.

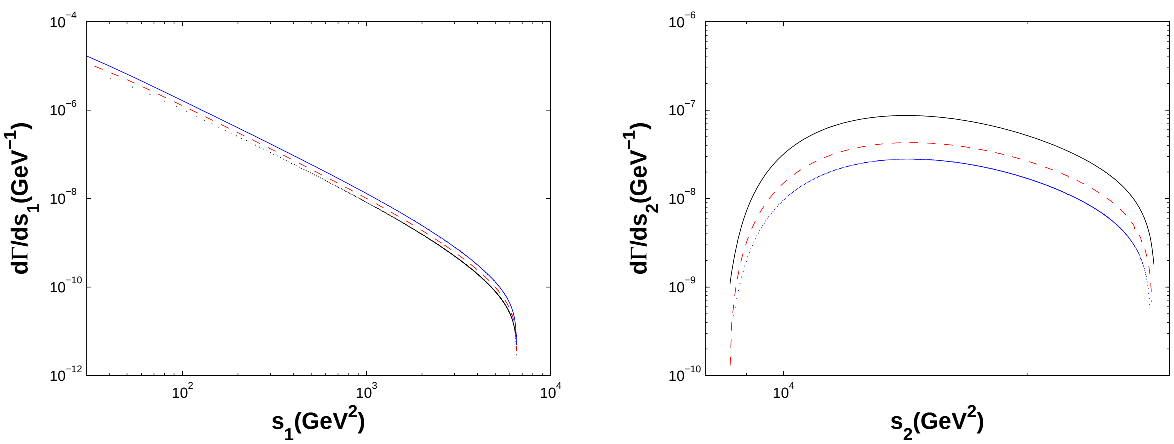

FIG. 6: Uncertainties of $d\Gamma/ds_1$ (Left) and $d\Gamma/ds_2$ (Right) for $t \to |(c\bar{c})[n]\rangle + cZ^0$, where the contributions from the color-singlet S-wave and P-wave states have been summed up. The solid, the dashed and the dotted lines are for $m_c = 1.25$ GeV, 1.50 GeV and 1.75 GeV, respectively.

To show how the decay widths depend on the quark masses, taking $t \to |(c\bar{c})[n]\rangle + cZ^0$ as an example, we present its differential decay width with several typical m_c and m_b in Figs.(6,7). Uncertainties of $d\Gamma/ds_1$ and $d\Gamma/ds_2$ of the process are presented in Fig.(6), where the contributions from the color-singlet S-wave and P-wave states have been summed up; Uncertainties of $d\Gamma/d\cos\theta_{13}$ and $d\Gamma/d\cos\theta_{12}$ of the process are presented in Fig.(7). In

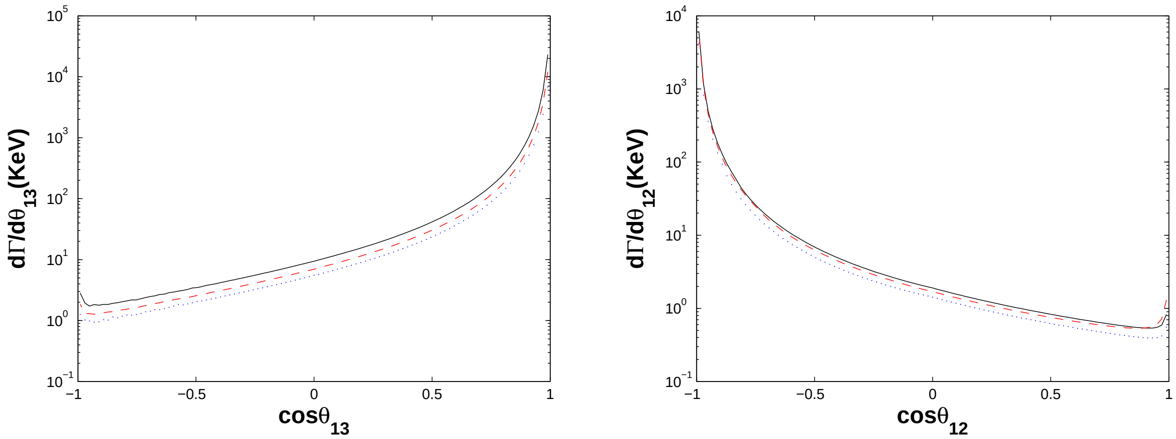

FIG. 7: Uncertainties of $d\Gamma/d\cos\theta_{13}$ (Left) and $d\Gamma/d\cos\theta_{12}$ (Right) for $t \to |(c\bar{c})[n]\rangle + cZ^0$, where contributions from the color-singlet S-wave and P-wave states have been summed up. The solid, the dashed and the dotted lines are for $m_c = 1.25$ GeV, 1.50 GeV and 1.75 GeV, respectively.

these figures, the contributions from the color-singlet S-wave and P-wave states have been summed up.

IV. CONCLUSIONS

Being by far the heaviest fundamental particle and the only quark decaying before hadronization takes place, the top quark provides a privileged tool for precise studies of the SM. A precise study of the top quark is helpful for understanding the hadronic and electroweak interactions and for searching new physics beyond the SM.

We have made a detailed study on the heavy-quarkonium production through the top quark semi-exclusive decays via FCNC, $t \to |(c\bar{Q})[n]\rangle + QZ^0$, within the NRQCD framework. Results for the eight quarkonium Fock states, i.e. $|(c\bar{Q})[^1S_0]_{\mathbf{1},\mathbf{8}}\rangle$, $|(c\bar{Q})[^3S_1]_{\mathbf{1},\mathbf{8}}\rangle$, $|(c\bar{Q})[^1P_1]_{\mathbf{1}}\rangle$ and $|(c\bar{Q})[^3P_J]_{\mathbf{1}}\rangle$ have been presented. To provide the analytical expressions as simplify as possible and to improve the numerical efficiency, we have adopted the 'New Trace Technology' for dealing with the hard scattering amplitude directly at the amplitude level.

Numerical results show that the color-singlet P-wave states in addition to the S-wave states can also provide sizable contributions to heavy quarkonium production, so one also needs to take the P-wave states into consideration for a sound estimation. For the charmonium production channel $t \to |(c\bar{c})[n]\rangle + cZ^0$, the total decay width for all the P-wave states

is ~ 27.2 KeV, which is about 39% (38%) of that of η_c (J/ψ). For the $(c\bar{b})$-quarkonium production channel $t \to |(c\bar{b})[n]\rangle + bZ^0$, the total decay width for all P-wave states is ~ 0.78 KeV, which is about 22% (26%) of that of B_c (B_c^*). If assuming the high excited heavy-quarkonium states decay to the ground color-singlet and spin-singlet S wave state $|(c\bar{Q})[^1S_0]_{\mathbf{1}}\rangle$ with 100% efficiency via the electromagnetic or the hadronic interactions, then we obtain the total decay width of top quark decay channels via FCNC

$$\begin{aligned}
\Gamma_{t\to|(c\bar{c})[^1S_0]_{\mathbf{1}}\rangle} &= 171.1^{+147.7}_{-68.8}\ \mathrm{KeV}, \\
\Gamma_{t\to|(c\bar{b})[^1S_0]_{\mathbf{1}}\rangle} &= 7.32^{+2.49}_{-1.75}\ \mathrm{KeV}.
\end{aligned}$$

At the LHC, due to its high collision energy and high luminosity, sizable heavy-quarkonium events can be produced through top quark decays via FCNC, i.e. about 10^4 η_c and 10^3 B_c events per year can be obtained.

Acknowledgements: This work was supported in part by the Fundamental Research Funds for the Central Universities under Grant No. CQDXWL-2012-Z002, the Program for New Century Excellent Talents in University under Grant No. NCET-10-0882, and the Natural Science Foundation of China under Grant No. 11075225 and No. 11275280.

Appendix A: $\mathcal{A}_n$ for $t \to (c\bar{Q})[n] + QZ^0$

For convenience, we present the $\mathcal{A}_n$ for the hard scattering of the processes $t(k) \to |(c\bar{Q})[n]\rangle(q_3) + Z^0(q_2) + Q(q_1)$ based on the Feynman diagrams shown in Fig.(1), where Q stands for c or b quark respectively.

For the $(c\bar{Q})$-quarkonium in S-wave states $\mathcal{A}_n$ can be written as

$$\mathcal{A}_1 = \left[\gamma_\mu \frac{\Pi^{0(\beta)}_{q_3}(q)}{(q_{32}+q_1)^2}\gamma_\mu \frac{\not{q}_1 + \not{q}_3 + m_c}{(q_1+q_3)^2 - m_c^2}\not{\epsilon}(q_2)(T_c - \gamma_5)\right]_{q=0}, \tag{A1}$$

$$\mathcal{A}_2 = \left[\gamma_\mu \frac{\Pi^{0(\beta)}_{q_3}(q)}{(q_{32}+q_1)^2}\not{\epsilon}(q_2)(T_c - \gamma_5)\frac{\not{q}_2 + \not{q}_{31} + m_t}{(q_2+q_{31})^2 - m_t^2}\gamma_\mu\right]_{q=0}. \tag{A2}$$

For the $(c\bar{Q})$-quarkonium in P-wave states $\mathcal{A}_n$ can be written as

$$\mathcal{A}_1^{S=0,L=1} = \varepsilon_l^\alpha(q_3)\frac{d}{dq_\alpha}\left[\gamma_\mu \frac{\Pi^{0}_{q_3}(q)}{(q_{32}+q_1)^2}\gamma_\mu \frac{\not{q}_1 + \not{q}_3 + m_c}{(q_1+q_3)^2 - m_c^2}\not{\epsilon}(q_2)(T_c - \gamma_5)\right]_{q=0}, \tag{A3}$$

$$\mathcal{A}_2^{S=0,L=1} = \varepsilon_l^{\alpha}(q_3)\frac{d}{dq_\alpha}\left[\gamma_\mu\frac{\Pi^0_{q_3}(q)}{(q_{32}+q_1)^2}\not\varepsilon(q_2)(T_c-\gamma_5)\frac{\not q_2+\not q_{31}+m_t}{(q_2+q_{31})^2-m_t^2}\gamma_\mu\right]_{q=0}, \tag{A4}$$

$$\mathcal{A}_1^{S=1,L=1} = \varepsilon^J_{\alpha\beta}(q_3)\frac{d}{dq_\alpha}\left[\gamma_\mu\frac{\Pi^\beta_{q_3}(q)}{(q_{32}+q_1)^2}\gamma_\mu\frac{\not q_1+\not q_3+m_c}{(q_1+q_3)^2-m_c^2}\not\varepsilon(q_2)(T_c-\gamma_5)\right]_{q=0}, \tag{A5}$$

$$\mathcal{A}_2^{S=1,L=1} = \varepsilon^J_{\alpha\beta}(q_3)\frac{d}{dq_\alpha}\left[\gamma_\mu\frac{\Pi^\beta_{q_3}(q)}{(q_{32}+q_1)^2}\not\varepsilon(q_2)(T_c-\gamma_5)\frac{\not q_2+\not q_{31}+m_t}{(q_2+q_{31})^2-m_t^2}\gamma_\mu\right]_{q=0}. \tag{A6}$$

Here $T_c = 1-\frac{8}{3}\sin^2(\theta_W)$. $\varepsilon(q_2)$ is the polarization vector of Z^0 boson. $\varepsilon_s(q_3)$ and $\varepsilon_l(q_3)$ are the polarization vectors relating to the spin and the orbit angular momentum of $(c\bar{Q})$-quarkonium, $\varepsilon^J_{\alpha\beta}(q_3)$ is the polarization tensor for the spin triplet P-wave states with $J=0,1$ and 2, respectively. q_{31} and q_{32} are the momenta of the two constitute quarks, i.e.

$$q_{31} = \frac{m_c}{M}q_3+q, \quad q_{32} = \frac{m_Q}{M}q_3-q. \tag{A7}$$

where q stands for the relative momentum between the two constitute quarks in $(c\bar{Q})$-quarkonium. $M\simeq m_c+m_Q$ are adopted to ensure the gauge invariance of the hard scattering amplitude. The projectors $\Pi^0_{q_3}(q)$ and $\Pi^\beta_{q_3}(q)$ are for spin-singlet and spin-triplet states accordingly, and their covariant form can be written as [41]

$$\Pi^0_{q_3}(q) = \frac{-\sqrt{M}}{4m_cm_Q}(\not q_{32}-m_Q)\gamma_5(\not q_{31}+m_c), \tag{A8}$$

$$\Pi^\beta_{q_3}(q) = \frac{-\sqrt{M}}{4m_cm_Q}(\not q_{32}-m_Q)\gamma_\beta(\not q_{31}+m_c). \tag{A9}$$

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